Compyter analysis of undulators with block-periodic stucture

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Abstract

Methods to detect the spectral sensitivity of an object using undulator radiation without monochromators or any other spectral devices are developed. The spectral transmission function of the object is calculated from its response to the spectrum-integral undulator radiation with the known spectral distribution. This response is measured as a function of the electron energy.

l. Introduction

At present, synchrotron radiation (SR) is widely used in spectroscopy as a standard source, the monochromatic components usually being obtained by mono- chromators or other spectral devices. However, a large intensity loss and variation of the transmission function during the operating time are inherent to such measurements.

Recently, undulator radiation (UR) has been discussed as an alternative. As is known [1], monochroma- tization of UR can be partially achieved by increasing the period number. UR serving as a standard source in spectroscopy without monochromators has been discussed in ref. [2]. It appears that the UR resolution is not only limited by the spectral line width that depends on the period number, but is also limited by the spread of angles and electron energies, the undulator magnetic field nonuniformity over the beam cross section, the finite diaphragm size, and other factors.

In this connection, a monochromatorless computer spectroscopy method (MCS method) has been proposed [3,4], the computer algorithm playing the part of the monochromator. The point is that the radiation from an undulator installed in the synchrotron ring is not pure UR as assume'd in the ideal theoretical model [1,5], but also includes the SR components from the edges of bending magnets and focusing elements adjacent to the undulator. In practice, one uses the frequency partition of SR and UR spectra to exclude the admixed SR and to conserve the ideal UR properties [6]. But we cannot admit this method to be consistent with the requirements of metrology, according to which all the ideal UR properties (angular monochromatization, polarization, independence of the

UR spectral form from the particle energy) need to be confirmed by adequate quantitative measurements.

2. Amplitude-time modulation

In the MCS method, the electron-energy-invariant spectral form of the. radiation source turns out to be a kernel of the integral equation, the solution of which being just the MCS problem. In order to make the UR kernel metrologically pure, we have to exclude the admixed SR. To this end we can use a procedure consistng of a series of consequent measurements of the undulator radiation at various states of the undulator magnetic system and the subsequent combination of these results. Such a procedure will be referred to as the amplitude - time modulation (ATM).

Parameters varying in the ATM process are those of the undulator, which do not disturb the properties of the admixed SR. This means that the phase relations for the SR components must be invariant during ATM. This implies the constancy of the time t for a charge travelling along a straight section of length l between the edge elements adjacent to the undulator:

$$t = \frac{1}{c}\Big[l + \frac{l + k^2 L}{2\gamma^2}\Big]. \tag{1}$$

Here L is the undulator length, γ the Lorenz factor of the charge ($\gamma^{-1} \ll 1$), c is the light velocity, and k is the undulator dipole parameter. The directional modulation of the magnetic field in the undulator or in some of its blocks is a special case of ATM since the electron transit time is conserved. The magnetic field in any block satisfies the balancing condition, i.e., the field integral over the block length vanishes.

Suppose $A(\omega)$ and $B(\omega)$ are the complex Fourier amplitudes of the ideal UR and the SR, respectively. Every $A(\omega)$ corresponds to one undulator magnetic field state. Let us consider the following four states: the basic state A; the state $(-A)$, which differs from A in that the undulator magnetic field is switched on with an orientation opposite to the field in the elements forming the electron orbit and adjoining the undulator; the phase-discontinuous state $\tilde{A}$ obtained by the reverse-sign switching of the magnetic field in some undulator blocks; and finally $(-\tilde{A})$. Adding the radiation intensi- ties in the first and second states and subtracting those in the third and the fourth states, we obtain

$$|A + B|^2 + |-A + B|^2 - |\tilde{A} + B|^2 - |-\tilde{A} + B|^2 = 2\big[|A|^2 - |\tilde{A}|^2\big]. \tag{2}$$

Thus, the four-step ATM with phase switching enables us to exclude the SR and to obtain the metrologically pure UR kernel for MCS as an element of the ideal UR (eq. (2)). The practical realization will be especially simple when one ATM step corresponds to one acceleration cycle. In this case, we need only four cycles to achieve a metrologically pure procedure with UR.

3. Undulator magnetic system

Among the various possible realizations of the undulator magnetic system for MCS with UR, the electromagnetic ironless system is preferable since only such a system has the desired properties, such as linearity, predictability, reiteration and the ability to change undulator states quickly. A block-periodic organization of the ironless electromagnetic undulator system in which the resulting distribution of the magnetic field is a superposition of fields from standard elements switching on mth given weights, seems to be the most suitable one for many types of ATM. In the general case, because of the common standard elements, the blocks may overlap one another with the equivalent summarized overlap weight.

The MCS technique needs no monochromatization devices since the computer algorithm plays the part of the monochromator. Thus we avoid the many order waste of the source intensity, which is usually inevitable with radiation monochromatization. It appears from this that an undulator with the dipole regime of UR excitation ($k \ll 1$) will be quite effective in achieving an adequate reaction of the object in the MCS technique. The dipole regime is favourable also with regard to the 'operation of the undulator magnetic system, since the decreased heat and electromagnetic loads permit the ironless variant of this system to be used.

Therefore, we can now regard the undulator dipole regime as the basic one for the MCS technique with UR, and so the model investigations of MCS with UR, based on the dipole approximation expression for UR, become legitimate.

In ATM it is important to provide the coincidence of the low-frequency trend components of the motion in the undulator for different steps of the ATM period. The amplitudes of the low-frequency components of the spectrum depend essentially on the values of the integrals

$$J_1 = \int_0^b H(x)dx, \quad J_2 = \int_0^b dx' \int_0^{x'} H(x)dx, \ ...$$

where $H(x)$ is the magnetic field along the undulator axis and b is the block length. We shall call the motion of a particle in a block an m-time balanced motion, when $J_1 = J_2 = ... = J_m = 0$. The Fourier structure of the UR line of a block with the motion balancing degree m contains the factor)

$$|\cos^m \bar{\omega} \sin \bar{\omega}(N-m)/sin\bar{\omega}|^2,$$

where $\bar{\omega} = \pi(\nu+1)/2$, $\nu = \eta(1+\psi^2)$ is the number of the UR harmonic at an angle θ to the motion axis, $\psi = \gamma\theta/\sqrt{1+k^2}, \eta = p\omega(1+k^2)/2\pi c\gamma^2, \omega$ is the radiation frequency, p is the magnetic field half-period length, and N is the number of standard elements in the block. The low-frequency asymptote for such a spectrum is ν^{2m} . It appears from this that in the region $0 < \nu < 1$ the UR spectral density at a given direction θ is suppressed and the number of spectral function zeros decreases when m increases. This causes

the oscillating part of the angle-integral UR spectrum to be depressed. As a result, the difference UR kernel of the type of eq. (2) formed by the ATM is localized in a region of the high-frequency cut-off of the fundamental harmonic. The maximal balancing degree in a block consisting of N standard elements equals $N-l$. In this case, the maximal smoothness of the UR integral spectrum and the maximal frequency, angle and polarization localization of the difference kernel of type (2) are obtained. The balancing degree desired is achieved by thc proper choice of the standard element weights. For example, for $m=1$ we have the standard element weight distribution $1,2,3,...,2,1$ and for $m=2$ we get $1,3,4,4,...,4,3,1$. With increasing m the weight distribution tends from a trapezoidal to a binomial one, the latter corresponding to $m=N-1$.

4. Basic equation

In the case where the object reaction depends lin early on the incident radiation amplitude, its respons to a part of the flux of the ideal UR (2) can be writte as follows:

$$J\Big(\frac{1+k^2}{\gamma^2}\Big)=\int\limits_0^\infty \frac{d\Phi_\Delta(\eta)}{d\omega'}\Pi(\omega)d\omega', \tag{3}$$

where

$$\frac{d\Phi_\Delta}{d\omega'}=\frac{d\Phi}{d\omega'}\Big|_A-\frac{d\Phi}{d\omega'}\Big|_{\tilde{A}},\quad d\omega'=\frac{d\omega}{\omega}.$$

$d\Phi/d\omega'\big|_{A,\tilde{A}}$ - is the integral spectral density of the photon flux for undulator magnetic field states A and $\tilde{A}$, $\Pi(\omega)$ is the spectral sensitivity of the object.

By the following change of variables in eq. (3):

$$\eta=e^\tau,\quad \omega=\frac{2\pi c}{p}e^{-s},\quad \frac{1+k^2}{\gamma^2}=e^x,$$

we obtain the standard formula for the object reaction to the UR as a convolution-type Fredholm integral eguation of the first kind:

$$U(x)=\int\limits_{-\infty}^{\infty} K(x-s)Z(s)ds. \tag{4}$$

Here,

$$U(x)=J(e^x),\quad K(\tau)=\frac{d\Phi_\Delta}{d\omega'}(e^\tau),\quad Z(s)=\Pi\Big(\frac{2\pi c}{p}e^{-s}\Big).$$

Eq. (4) is a basic one in the MCS problem. In order to solve this problem the object reaction is measured as a function of the particle energy. The speetral density $K(\tau)$ of the UR photon flux is measured experimentally or calculated using known formulas. The spectral sensitivity of the object $Z(s)$ is found

from eq. (4) by numerical calculations using the regularization methods which are applied to solve ill-defined problems. The angle-integral density of the UR photon flux can be written down in the dipole approximation [5,7] for σ- and π-polarization components:

$$\frac{d\Phi}{d\omega'}\Big|_{\pi}^{\sigma} = \frac{4\pi L\alpha k^2}{3Tp}\frac{I_1}{I_2}\phi(\eta)\Big|_{\pi}^{\sigma},$$

$$\phi(\eta)\Big|_{\pi}^{\sigma} = \frac{4}{3I_1}\eta\int\limits_{\eta}^{\infty}|H(\omega_x)|^2\left[\begin{array}{c} 1-2\frac{\eta}{\nu}+3\big(\frac{\eta}{\nu}\big)^2 \\ 1-2\frac{\eta}{\nu}+\big(\frac{\eta}{\nu}\big)^2 \end{array}\right]\frac{d\nu}{\nu^2},$$

with the normalizing condition

$$\int\limits_0^{\infty}\big[\phi(\eta)\big|_{\sigma}+\phi(\eta)\big|_{\pi}\big]\frac{d\eta}{\eta}=1.$$

Here $\alpha\approx 1/137$ is the fine structure constant, T is the orbit period for a charge in an accelerator or storage ring,

$$I_j=\int\limits_0^{\infty}\big|H(\omega_x)\big|^2\frac{d\nu}{\nu^j},\quad H(\omega_x)=\int\limits_{-\infty}^{\infty}H(x)e^{i\omega_x x}dx,\quad k^2=\Big(\frac{\mu e_0}{\pi m_0 c}\Big)^2\frac{p}{L}I_2,$$

is the dipole parameter, e_0 is the electron charge, m_0, is the electron mass, $\omega_x=\pi\nu/p,\ \ \mu=4\pi\cdot 10^{-7}\ H/m$. In the numerical experiment on the ATM with phase reversal and weight modulation we have used the wide undulator approximation which is close to that in practice, the field being dependent only on x. The phase modulation has been realized by alternately switching of the polarity of the undulator block supplies. The criterion for the choice of the correct modulation parameters $\{N,\ n,\ m,\ D/p\}$ is the vanishing integral difference given by:

$$\Delta=\int\limits_0^{\infty}\frac{d\Phi_{\Delta}}{d\omega'}(\eta)\frac{d\eta}{\eta}=\frac{4\pi\alpha}{3T}\Big(\frac{\mu e_0}{\pi m_0 c}\Big)^2\int\limits_0^{\infty}\big[|H(\omega_x)|_A^2-|H(\omega_x)|_{\bar{A}}^2\big]\frac{d\nu}{\nu},\qquad(5)$$

where n is the number of blocks and D is the distance between the neighbouring blocks. The geometric param- eters of the standard elements and their number M are found from the basic initial parameters, namely, the undulator length and the undulator gap. Knowing M one can start a model run in order to find the remaining parameters $\{N,\ n,\ m,\ D/p\}$. If the neighbouring blocks overlap, then $D<0,\ D/p$ being an integer. The qualitative criteria for the results of this run are: (i) the vanishing Δ in eq. (5) with $D/p<0$; (ii) the correctness of the solution of the MCS problem with the obtained UR kernel.

5. Conclusions

(1) When the motion balancing degree m increases, an effective depression of the low-frequency component of the UR kernel $K(x-s)$ in the region $x-s<0$ takes place in the undulator blocks.

(2) When m increases, localization of the UR kernel in the region $\eta \approx 1$ of the high-frequency cut-off of the total ideal kernel occurs. This monochromatization effect corresponds to the angular localization of the UR kernel in the angular range $\psi \in (0,\ 1/\sqrt{N})$, in accordance with the known formula $\nu = \eta(1+\psi^2)$. This angular localization near the direction $\psi = 0$ causes the preferable selection of the σ-component of UR polarization and the strong depression of the π-component, since the intensity of the π-component is small for directions $\psi < 1\sqrt{N}$.

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